REPORT DOCUMENTATION PAGE

AFRL-SR-AR-TR-04-

0204

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE March 9, 2004	3. REPORT TYPE AND DATES COVERED Final Performance Report 7/1/03-12/31/03

4. TITLE AND SUBTITLE Development of MHD Algorithms on Type II Quantum Computers	5. FUNDING NUMBERS G F49620-01-1-0462
6. AUTHOR(S) Dr. Linda Vahala	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Old Dominion University Research Foundation P O Box 6369 Norfolk VA 23508	8. PERFORMING ORGANIZATION REPORT NUMBER 213971
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) USAF, AFRL Air Force Office of Scientific Research 801 N Randolph St Arlington VA 22203-1977	10. SPONSORING / MONITORING AGENCY REPORT NUMBER
11. SUPPLEMENTARY NOTES	
12 a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution unlimited.	12 b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

While a classical computer manipulates either a "0" or "1" bit, a quantum computer can manipulate states that are arbitrary superpositions of these base states-a qubit state. Quantum computers can entangle and operate on a collection of qubits in parallel and thereby provide exponential speed up over classical computers. Quantum lattice algorithms have been developed to solve problems that are very difficult to solve in classical computers-e.g. magnetohydrodynamic turbulence. In particular a special sequence of collide-steam operators, that can be implemented on a quantum computer, has been devised that will recover the one-dimensional magnetohydrodynamic equations. The results are compared to classical algorithms and the results are in excellent agreement. The power law spectrum is shown to be k^{-2}. Quantum lattice algorithms have also been developed for nonlinear Schrodinger equations and Korteveg-deVries solitons. There is again excellent agreement between the quantum algorithm solutions and the exact analytic soliton solutions.

20040423 018

14. SUBJECT TERMS	15. NUMBER OF PAGES 2
	16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION ON THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL

Final AFOSR Report

"Development of MHD Algorithms on Type II Quantum Computers"

Linda Vahala

Old Dominion University
Norfolk, VA 23529

We began research on developing quantum lattice algorithms in the Spring of 2001. During the period of this AFOSR grant we have tested the accuracy of our quantum algorithms on exact soliton solutions to the Nonlinear Schrodinger equation (NLS) as well as to 1D MHD turbulence. The following papers have been published and/or submitted for publication under the AFOSR grant:

Lattice Boltzmann and Quantum Lattice Gas Representations of One-Dimensional Magnetohydrodynamic Turbulence
L. Vahala, G. Vahala and J. Yepez
Phys. Lett. **A306**, 227-234 (2003)

Quantum Lattice Gas Representation of Some Classical Solitons
G. Vahala, J. Yepez and L. Vahala
Phys. Lett. **A310**, 187-196 (2003)

Quantum lattice gas representation for vector solitons
G. Vahala, L. Vahala, and J. Yepez
SPIE Conf. Proc. **5105,** 273 – 281 (2003)

Inelastic Vector Soliton Collisions: A Quantum Lattice Gas Representation
G. Vahala, L. Vahala and J. Yepez
Phil. Trans.. Roy Soc. London (accepted for publication, 2004)

Quantum lattice representation of dark solitons
G. Vahala, L. Vahala, and J. Yepez
SPIE Conf. Proc. **5436,** (to be published, 2004)

As insights into fluid turbulence has been gained by examining simplified nonlinear one-dimensional (1D) models as Burgers equation. Since MHD turbulence involves more complex structures, Yanase (1997) and others have developed a 1D magnetized generalization of the Burgers equation. In its simplest form, this self-consistent model is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{B^2}{2}\right) + \mu\frac{\partial^2 u}{\partial x^2} \quad , \quad \frac{\partial B}{\partial t} + \frac{\partial}{\partial x}(uB) = \eta\frac{\partial^2 B}{\partial x^2} \tag{1}$$

where the fluid velocity $\mathbf{u} = u(x,t)\hat{x}$ and the magnetic field $\mathbf{B}=B(x,t)\ \hat{z}$. μ is the viscosity and η is the resistivity. This 1D MHD model reduces to the regular Burgers equation in the limit of zero magnetic field : $B\to 0$. However, for non-zero magnetic fields one notes that there is a self-consistent magnetic pressure that mitigates to reduce the Burger shock formations in the fluid velocity. Moreover this 1D model exhibits rugged invariants as the full MHD equations : i.e., in the inviscid limit $\mu\to 0, \eta\to 0$ the invariants are the energy and cross helicity :

$$E_K + E_M = \frac{1}{2}\int dx\left(u^2+B^2\right) \quad , \quad E_{CH} = \int dx\, uB \tag{2}$$

In turbulence, it is critical to accurately compute energy transfer rates. For 1D MHD, the energy transfer from the magnetic to velocity field is given by

$$T_{B\to u} = \frac{1}{2}\int dx \frac{\partial u}{\partial x} B^2$$

with the kinetic and magnetic energy transfer equations

$$\frac{\partial E_K}{\partial t} = T_{B\to u} - 2\mu Q_K \quad , \quad \frac{\partial E_M}{\partial t} = -T_{B\to u} - 2\eta Q_M \qquad (3)$$

where $Q_K = \frac{1}{2}\int dx \left(\frac{\partial u}{\partial x}\right)^2 \quad , \quad Q_B = \frac{1}{2}\int dx \left(\frac{\partial B}{\partial x}\right)^2$.

To develop a quantum lattice representation for this 1D MHD model we first discretize the spatial dimension and introduce 2 qubits/scalar field at each node. To prepare the initial state one considers each qubit to have the form

$$|q_a(x)\rangle = \sqrt{p_a(x)}\,|1\rangle + \sqrt{1-p_a(x)}\,|0\rangle \quad , \quad a = 1\ldots4$$

where p_a is the probability that the excited state is occupied. From the given initial velocity and magnetic field profiles, one initializes the excited state probabilities

$$p_1 = \frac{1}{2}(1+u+B) = p_3 \quad , \quad p_2 = \frac{1}{2}(1+u-B) = p_4$$

One now applies a local unitary collision interaction $\hat{U}$ to entangle the on-site qubits

$$|\psi'(x)\rangle = \hat{U}|\psi(x)\rangle \; ,$$

where the pre-collision on-site ket $|\psi(x)\rangle = |q_1(x)\rangle \otimes |q_2(x)\rangle \otimes |q_3(x)\rangle \otimes |q_4(x)\rangle$.
On these post-collision probabilities one performs a nonunitary measurement that destroys the quantum entanglement introduced by the collision operator:

$$p_a'(x) = \langle\psi'(x)|\hat{n}_a|\psi'(x)\rangle$$

Finally, one streams these probabilities p_a' to nearest neighbor nodes.
Thus the final kinetic equation for the post-collision probabilities

$$p_a(x+e_a,t+1) = p_a(x,t) + \langle\psi(x)|\,\hat{U}^{+}\hat{n}_a\hat{U} - \hat{n}_a\,|\psi'(x)\rangle$$

To recover the 1D MHD equations, one first entangles qubits '3' and '4' with the square-root-of-swap collision operator

$$\hat{U}_{\sqrt{swap}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and then streams these post-collision qubit states '3' and '4' to nearby sites

$$p_a^{(1)}(x+e_a,t+1) = p_a^{(1)}(x,t) + \langle\psi(x)|\,\hat{U}^{(3,4)+}_{\sqrt{swap}}\hat{n}_a\hat{U}^{(3,4)}_{\sqrt{swap}} - \hat{n}_a\,|\psi'(x)\rangle \quad , \quad e_a = (0,0,1,-1)$$

One now reinitializes the occupation probabilities

$$p_1^{(1)} + p_2^{(1)} + p_3^{(1)} + p_4^{(1)} = p_1^{(2)} = p_2^{(2)} \; , \; p_1^{(1)} + p_2^{(1)} - p_3^{(1)} - p_4^{(1)} = p_3^{(2)} = p_4^{(2)}$$

and then entangles qubits '1'-'2' and qubits '3'-'4' followed by appropriate streaming to nearby spatial nodes. Thus

$$p_a^{(2)}(x+e_a,t+1) = p_a^{(2)}(x,t) + \langle\psi(x)|\,\hat{U}_{\pi/4}^{(3,4)+}\hat{U}_{\pi/4}^{(1,2)+}\,\hat{n}_a\hat{U}_{\pi/4}^{(3,4)}\,\hat{U}_{\pi/4}^{(1,2)} - \hat{n}_a\,|\psi'(x)\rangle,$$

with streaming $e_a = (1,-1,1,-1)$ and the $\pi/4$ - unitary rotation matrix

$$\hat{U}_{\pi/4} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

One identifies the velocity and magnetic fields from the final occupation probabilities

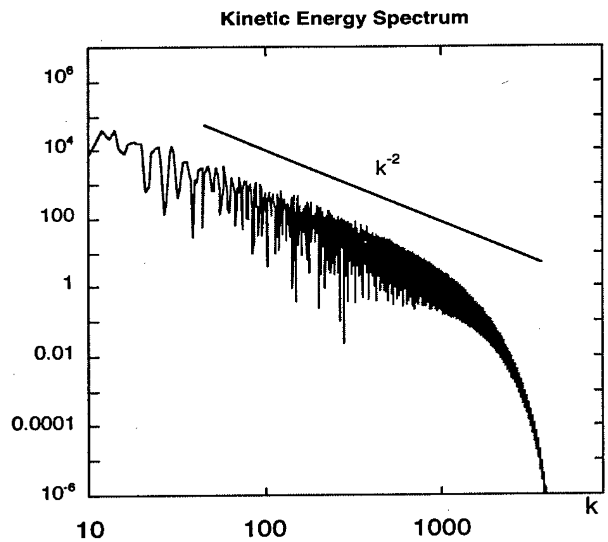
$$\begin{aligned} 2u(x,t) &= p_1^{(2)}(x,t) + p_2^{(2)}(x,t) + p_3^{(2)}(x,t) + p_4^{(2)}(x,t) \\ 2B(x,t) &= p_1^{(2)}(x,t) + p_2^{(2)}(x,t) - p_3^{(2)}(x,t) - p_4^{(2)}(x,t) \end{aligned} \tag{4}$$

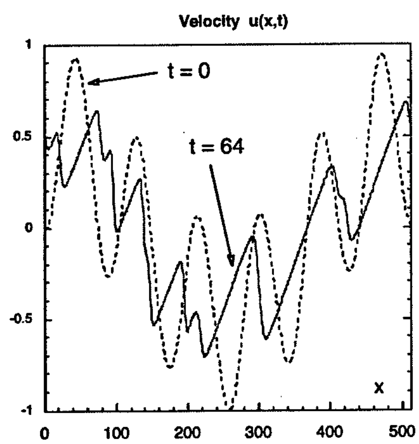
and the unconditionally stable explicit finite difference scheme that emerges from this quantum lattice algorithm (in lattice units $\Delta x = 1 = \Delta t$)

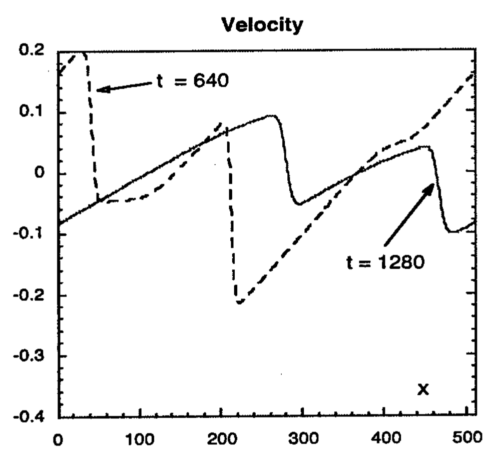
$$\begin{aligned} u(x,t+1) &= \frac{1}{2}\left[u(x-1,t)+u(x+1,t)\right] + \frac{1}{4}\left[u^2(x-1,t)-u^2(x+1,t)\right] + \\ &\quad \frac{1}{16}\left[B(x-2,t)-B(x+2,t)\right]\left[B(x-1,t)+2B(x,t)+B(x+1,t)\right] \\ B(x,t+1) &= \frac{1}{4}\left[B(x-2,t)+2B(x,t)+B(x+2,t)\right] + \\ &\quad \frac{1}{4}\left[u(x-1,t)B(x-2,t)+\{u(x-1,t)-u(x+1,t)\}B(x,t) - u(x+1,t)B(x+2,t)\right] \end{aligned} \tag{5}$$

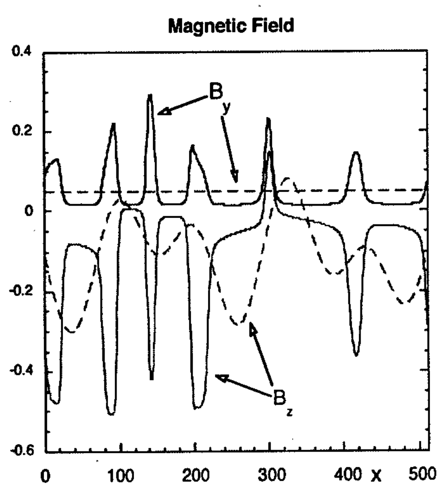
In the continuum limit, Eq. (5) reduces to the desired 1D MHD equations (1) with transport coefficients $\mu = \frac{(\Delta x)^2}{2\Delta t}$, $\eta = 2\mu$. This finite difference scheme can be immediately extended from a scalar to vector magnetic field $B \rightarrow \mathbf{B} = (0,B_y,B_z)$.

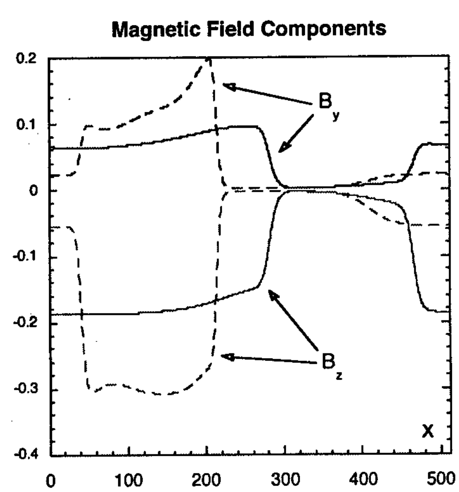
A typical simulation is shown below. The initial oscillatory velocity field (dashed curves) steepen following a Burgers-like shock development, but energy is transferred to the magnetic field so that these field are amplified in the regions of the shocks and the velocity shocks. The kinetic energy spectrum exhibits the k^{-2} spectrum.


Kinetic Energy Spectrum
10^6
10^4
100
1
0.01
0.0001
10^{-6}
10
100
1000
k
k^{-2}


Velocity u(x,t)
t = 0
t = 64
x
1
0.5
0
-0.5
-1
0
100
200
300
400
500


Velocity
t = 640
t = 1280
x
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
100
200
300
400
500


Magnetic Field
B_y
B_z
0.4
0.2
0
-0.2
-0.4
-0.6
0
100
200
300
400
x
500


Magnetic Field Components
B_y
B_z
x
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
100
200
300
400
500